

Optical Nondestructive Controlled-NOT Gate without Using Entangled Photons

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We present and experimentally demonstrate a novel optical nondestructive controlled-NOT gate without using entangled ancilla. With much fewer measurements compared with quantum process tomography, we get a good estimation of the gate fidelity. The result shows a great improvement compared with previous experiments. Moreover, we also show that quantum parallelism is achieved in our gate and the performance of the gate can not be reproduced by local operations and classical communications.

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The controlled-NOT (CNOT) or similar entangling gates between two individual quantum bits (qubits) are essential for quantum computation [1, 2]. Also entangling gates can be utilized to construct a complete Bell-state analyzer which is required in various quantum communication protocols [3, 4, 5]. Photons are one of the best candidates for qubit due to the robustness against decoherence and ease of single-qubit operation. So far there have been several experiments implementing the optical CNOT gate [6, 7, 8, 9, 10, 11, 12]. These experiments can be divided into two groups, one is the destructive CNOT gate [6, 7, 8, 9, 10] which means that one have to measure the output of the gate to verify a successful operation, imposing a great limitation for its further implementations, and the other is the nondestructive gate [11, 12].

For a nondestructive CNOT gate, the information whether the operation succeeds or not is provided. This information can then be utilized for future conditional operations on the photonic qubits to achieve efficient linear optical quantum computation. Also with this information arbitrary entangled state can be constructed in an efficient way, especially the cluster state for one-way quantum computation [13, 14]. So nondestructive CNOT gate is much more important than the destructive one. To build a nondestructive gate, usually ancilla photons are unavoidably required. Previous scheme [15] requires an entangled photon pair as assistance. The well developed SPDC (spontaneous parametric down-conversion) [16] entangled photon source will be unsuitable due to the probabilistic character. Generating entangled photons directly from quantum dots [17, 18] is still at its beginning and the fidelity is to be improved. Making use of entangled photons generated from single photons [19, 20, 21] is another solution, but it will make the setup much more complicated and reduce the success probability a lot under the present technology. Also the imperfections of the entangled photon pair will cause a degradation to the fidelity of the gate, making high-precision gate operation even more difficult to achieve.

Here we present and experimentally demonstrate a novel nondestructive CNOT gate without using entangled photons but only single photons instead, which is a great improvement compared with former scheme [15]. With much fewer measurements compared with quantum process tomography [22], we get a good estimation of the gate fidelity with the method developed by Hofmann in [23, 24].

In our scheme, the qubit we consider refers to the polarization state of photons. We define the polarization state $|H\rangle$ as logic 0 and $|V\rangle$ as logic 1. Let's assume that the input state of the control qubit is $|\psi\rangle^c = \alpha|H\rangle + \beta|V\rangle$ and of the target qubit is $|\psi\rangle^t = \gamma|H\rangle + \delta|V\rangle$. As shown in Fig. 1(a), two auxiliary photons with polarization state of $1/\sqrt{2}(|H\rangle + |V\rangle)$ and $|H\rangle$ are required. Then the total state of the input four photons can be expressed as:

$$|\Psi\rangle_1 = \frac{1}{2}(\alpha|H\rangle_c + \beta|V\rangle_c)(|H\rangle_{a1} + |V\rangle_{a1})(|+\rangle_{a2} + |-\rangle_{a2})(\gamma|H\rangle_t + \delta|V\rangle_t) \quad (1)$$

where the subscript (c, a1, a2 and t) represents each path of the four photons, state $|+\rangle$ equals to $1/\sqrt{2}(|H\rangle + |V\rangle)$ and $|-\rangle$ equals to $1/\sqrt{2}(|H\rangle - |V\rangle)$. First the four photons transmit through PBS-1 which transmits state $|H\rangle$ and reflects state $|V\rangle$ and PBS-2 which transmits state $|+\rangle$ and reflects state $|-\rangle$. Let's consider the case that there is one photon in each output path. Then the four-photon state will change to

$$|\Psi\rangle_2 = [\alpha|H\rangle_1|H\rangle_2 + \beta|V\rangle_1|V\rangle_2] \otimes [(\gamma + \delta)|+\rangle_3|+\rangle_4 + (\gamma - \delta)|-\rangle_3|-\rangle_4] \quad (2)$$

with a probability of 1/4. This state expanded in the Bell basis of photon 2 and photon 3 is shown as follows:

$$|\Psi\rangle_2 = I_1 I_4 U_{14} |\psi\rangle_1^c |\psi\rangle_4^t \otimes |\Phi^+\rangle_{23} + I_1 \sigma_{x4} U_{14} |\psi\rangle_1^c |\psi\rangle_4^t \otimes |\Psi^+\rangle_{23} + \sigma_{z1} I_4 U_{14} |\psi\rangle_1^c |\psi\rangle_4^t \otimes |\Phi^-\rangle_{23} + \sigma_{z1} \sigma_{x4} U_{14} |\psi\rangle_1^c |\psi\rangle_4^t \otimes |\Psi^-\rangle_{23} \quad (3)$$

where U refers to the CNOT operation; $|\Phi^\pm\rangle$ and $|\Psi^\pm\rangle$ are standard Bell states in $|H\rangle/|V\rangle$ basis; σ_x and σ_z are

Pauli operators with the form $\sigma_x = |H\rangle\langle V| + |V\rangle\langle H|$, $\sigma_z = |H\rangle\langle H| - |V\rangle\langle V|$.

From Equ. 3 we can see that if the jointly measured result of photon 2 and photon 3 is the state $|\Phi^+\rangle$, then the state of photon 1 and photon 4 is exactly the output state of the CNOT operation; if the measured result is other state ($|\Phi^-\rangle$, $|\Psi^+\rangle$ or $|\Psi^-\rangle$), then corresponding single qubit operations on the state of photon 1 and photon 4 are required to get the result of the CNOT operation. But within the linear optical technology only two of the four bell states can be distinguished. In our scheme as shown in Fig. 1(a), the two Bell states are $|\Phi^-\rangle$ and $|\Psi^+\rangle$. $|\Phi^-\rangle$ corresponds to the coincidence between D_A^H and D_B^H or between D_A^V and D_B^V and $|\Psi^+\rangle$ corresponds to the coincidence between D_A^H and D_B^V or between D_A^V and D_B^H . In conclusion if there is a coincident count between D_A (D_A^H or D_A^V) and D_B (D_B^H or D_B^V), then one bit of classical information will be sent to do the corresponding single qubit operation as shown in Fig. 1(a), and after that the state of photon 1 and photon 4 will be the exact output state of the CNOT operation. The total success probability is $1/8$.

For each PBS (PBS-1 and PBS-2) the output can be divided into three cases: one in each output path; two in first path and zero in the second; zero in the first and two in the second. Consider PBS-1 and PBS-2 jointly, there will be nine cases as follows:

$$\begin{aligned} \text{Group 1 } & 1 : 1 : 1 : 1 \\ \text{Group 2 } & 1 : 1 : 2 : 0 \quad 1 : 1 : 0 : 2 \quad 2 : 0 : 1 : 1 \\ & 0 : 2 : 1 : 1 \quad 2 : 0 : 0 : 2 \quad 0 : 2 : 2 : 0 \\ \text{Group 3 } & 2 : 0 : 2 : 0 \quad 0 : 2 : 0 : 2 \end{aligned}$$

where $n_1 : n_2 : n_3 : n_4$ corresponds to the photon numbers in each path (1, 2, 3 or 4). Group 1 is what we expected, just as what we have discussed. In group 2 the total number of photons on the path 2 and path 3 does not equal to 2, so the cases in this group will not give a correct trigger signal with assistance of photon number resolving detectors [25]. For the cases in group 3, the total photon number of path 2 and path 3 equals to 2. Roughly thinking, these two cases will lead to a coincidence between D_A and D_B , which will ruin this scheme. But considering the photon bunching effect [26], we will find that it is not possible for a correct trigger signal, because two photons either one in $|H\rangle$ and the other in $|V\rangle$ or one in $|+\rangle$ and the other in $|-\rangle$ will go to the same output path when they pass through a PBS in the R/L basis (PBS-3 in Fig. 1), where $|R\rangle = 1/\sqrt{2}(|H\rangle + i|V\rangle)$ and $|L\rangle = 1/\sqrt{2}(|H\rangle - i|V\rangle)$.

Our scheme works ideally with true single-photon input. But at present manipulating multi single photons simultaneously is still a difficult task [27]. In our proof-of-principle experiment, we utilized disentangled photons from SPDC [16] sources as the four input photons of the CNOT gate as shown in Fig. 1(b). Perfect spacial and temporal overlap on the three PBS are necessary, which

is highly related to the fidelity of the gate. In the experiment narrow-band interference filters are added in front of each detector to define the exact spectral emission mode, resulting in a coherence time longer than the pulse duration. All the photons are collected with single-mode fibers to define the exact spacial mode. Additional translators are added in path a1 and a2 to achieve good temporal overlap on PBS-1 and PBS-2. Previously to get the perfect temporal overlap between photons from different pairs on PBS-3, people have to measure the four-fold coincident counts as a function of scanning position of the delay mirror (as shown in Fig. 1(b)). But usually the four-fold count rate is very low (at the order of 1/sec typically), which usually makes the scanning process take a long time. In our experiment, we can overcome this difficulty by utilizing the two-photon Mach-Zehnder interference effect as shown in Fig. 2. As a result we can scan two-fold coincident counts instead, which is much higher than four-fold coincident counts, and this greatly shortens the process to get photons from different pairs temporal overlapped, usually we can find the temporal overlap on PBS-3 in about several tens of seconds only.

To evaluate the performance of our gate, first we test the capability to generate entanglement. We choose the input product state as $|+\rangle_c|H\rangle_t$. Corresponding to the CNOT operation, ideally the output state should be $|\Phi^+\rangle_{14}$, which is a maximal entangled state. To verify this, we measure the correlation between the polarizations of photon 1 and photon 4, and the measured visibilities are $(83.8 \pm 5.5)\%$ and $(96.0 \pm 2.8)\%$ for $|H\rangle/|V\rangle$ and $|+\rangle/|-\rangle$ basis, respectively. As we know for states with a visibility above 71%, Bell inequalities [28, 29] can be violated, which is a important criterion for entanglement.

In order to get the most complete and precise evaluation of a gate, previously quantum process tomography [22] has been utilized in former experiments [8, 9, 30]. However, 256 different measurement setups are required to evaluate only a CNOT gate. In contrast, here we utilize a recently proposed method [23] to fully evaluate our gate, in which only 32 measurements are required. From these measurements we can get the upper and lower bound of the gate fidelity.

As we know, in the computational basis ($|H\rangle/|V\rangle$) under the CNOT operation, the target qubit flips when the control qubit is logic 1 (state $|V\rangle$). However, this process gets reversed in the complementary basis ($|+\rangle/|-\rangle$) that the control qubit will flip if the target qubit is logic 1 (state $|-\rangle$). Measurement of the logic functions in these two bases will give a good estimation of the range of the gate fidelity. The experimental results are shown in Fig. 3(a) and Fig. 3(b). Let's define the fidelities in these two

bases as

$$\begin{aligned} F_1 &= 1/4 [P(HH|HH) + P(HV|HV) \\ &\quad + P(VV|VH) + P(VH|VV)] \\ F_2 &= 1/4 [P(++|++) + P(--|--) \\ &\quad + P(+-|-+) + P(+ -|- -)] \end{aligned} \quad (4)$$

where each P represents the probability to get the corresponding output state under the specified input state condition. In order to convert the coincident count rates to probabilities, we normalize them with the sum of coincidence counts obtained for the respective input state. In our experiment measured F_1 is $(88 \pm 1)\%$ and F_2 is $(90 \pm 1)\%$. As discussed in detail in Ref. [23], the upper bound and low bound of the gate fidelity can be obtained from these two fidelities as follows:

$$(F_1 + F_2 - 1) \leq F \leq \min(F_1, F_2). \quad (5)$$

In our experiment the lower and upper bounds of the gate fidelity are $(78 \pm 2)\%$ and $(88 \pm 1)\%$ respectively. Consider into the imperfections of the polarizers and waveplates used and the slightly higher order events (estimated ratio of 6-photon count rate to 4-photon count rate is only about 0.008), the fidelity of initial state preparation can be better than 98.9%. If the initial state preparation is perfect, the measured gate fidelity will be improved a little bit. We think that most of the degradation of the fidelity is due to the imperfection of PBS and the imperfect overlapping on it.

Recently a new experimental criterion for the evaluation of device performance has been proposed[24]. It was shown that a quantum controlled-NOT gate simultaneously performs the logical functions of three distinct conditional local operations. Each of these local operations can be verified by measuring a corresponding truth table of four local inputs and four local outputs. Specifically, quantum parallelism is achieved if the average fidelity of the three classical operations exceeds $2/3$. As a matter of fact the fidelity F_1 and F_2 are just two of the required three fidelities. The third fidelity is defined as

$$\begin{aligned} F_3 &= 1/4 [P(RL/+H) + P(LR/+H) \\ &\quad + P(RR/+V) + P(LL/+V) + P(RR/-H) \\ &\quad + P(LL/-H) + P(RL/-V) + P(LR/-V)] . \end{aligned} \quad (6)$$

The experimental result of F_3 is shown in Fig. 3(c) with the measured value $(90 \pm 1)\%$. The average fidelity of F_1 , F_2 and F_3 is $(89 \pm 1)\%$, exceeding the boundary $2/3$, which shows that quantum parallelism of our CNOT gate has been achieved and the performance of the gate can not be reproduced by local operations and classical communications.

In summary, we have presented and experimentally demonstrated a novel scheme to realize the optical non-destructive CNOT gate without using entangled photons but only single photons instead. With much fewer measurements compared with quantum process tomography

[22], we got a good estimation of the gate fidelity (between $(78 \pm 2)\%$ and $(88 \pm 1)\%$), showing a great improvement compared with previous experiments (In [12] severe noise from unwanted two-pair events has been subtracted from the experiment result; in [11] five photons were involved to avoid two-pair events, resulting in rather low visibility). Moreover, we have also shown that quantum parallelism was achieved in our CNOT gate. We believe that our experiment and the methods developed in this experiment would have various novel applications in the fields of both linear optical quantum information processing and quantum communication with photons.

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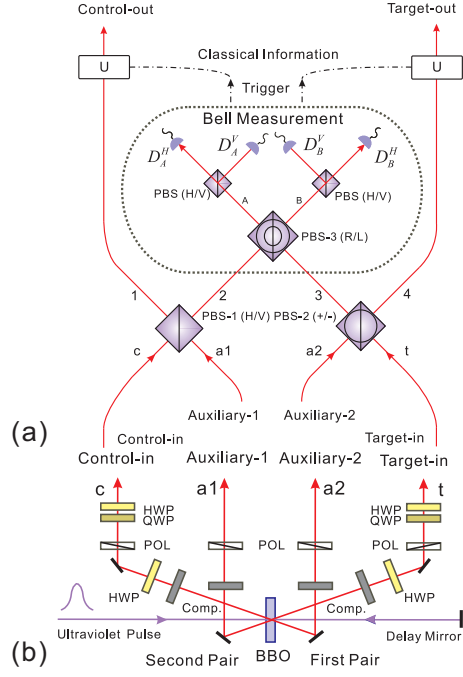


FIG. 1: (color online). (a) Our scheme to implement nondestructive CNOT gate with polarization beam splitters (PBS) in $|H\rangle/|V\rangle$ basis, in $|+\rangle/|-\rangle$ basis and in $|R\rangle/|L\rangle$ basis. PBS in $|+\rangle/|-\rangle$ basis (PBS-2) is constructed with a PBS in $|H\rangle/|V\rangle$ basis and four half-wave plates (HWP); PBS in $|R\rangle/|L\rangle$ basis (PBS-3) is constructed with a PBS in $|H\rangle/|V\rangle$ basis and four quarter-wave plates (QWP). This gate works like this: four photons (control qubit, target qubit and two auxiliary qubit) enter from the bottom; if there is a coincident count between detector D_A and detector D_B , a successful CNOT gate operation will be made after sending one bit classical information and doing the corresponding single-qubit unitary operations on photon 1 and 4. Then the state of photon 1 is exactly the output of the control qubit; and the state of photon 4 is exactly the output of the target qubit. In our proof-of-principle experiment, for simplification only the coincident events between D_A^H and D_B^H are registered, and a HWP is added to do the corresponding σ_z operation on photon 1. (b) Experimental setup to generate the required four photons. Near infrared femtosecond laser pulses (≈ 200 fs, 76 MHz, 788 nm) are converted to ultraviolet pulses through a frequency doubler LBO (LiB_3O_5) crystal (not shown). Then the ultraviolet pulse transmits through the main BBO ($\beta - BaB_2O_4$) crystal (2mm) generating the first photon pair, then reflected back generating the second photon pair. Compensators (Comp.) which is composed of a HWP(45°) and a BBO crystal (1mm) are added in each arm. The observed 2-fold coincident count rate is about $1.2 \times 10^4/s$. In each arm we add a polarizer to do the disentanglement and set the initial product four-photon state to $|H\rangle_c|+\rangle_{a1}|H\rangle_{a2}|H\rangle_t$. Additional wave plates are added in path c and path t to prepare arbitrary polarization states.

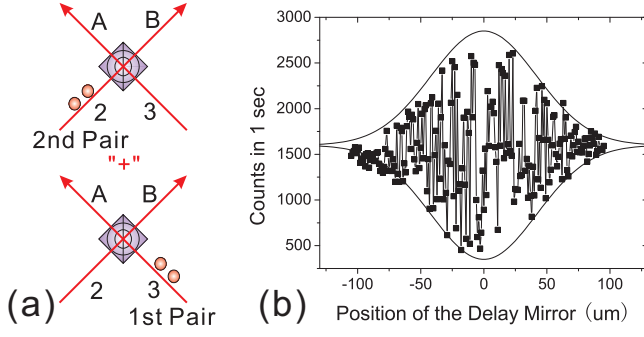


FIG. 2: (color online). Two-photon Mach-Zehnder interference as a method to find the overlap on PBS-3. (a) Schematic diagram of the method. After perfect overlaps on PBS-1 and PBS-2 have been achieved, by adjusting polarizers and wave plates, two photons originated from the first pair will be on path 3 with polarization state of $|1R, 1L\rangle_3$ and the other two photons originated from the second pair will be on path 2 with polarization state of $|1R, 1L\rangle_2$. Consider that the probability of generating two pairs simultaneously is rarely low, so the coincident click between detector D_A and detector D_B maybe originate either from the two photons on path 2 or from the two photons on path 3. So the two-photon state before PBS-3 can be expressed as $1/\sqrt{2}(|1R, 1L\rangle_3 + e^{i\phi}|1R, 1L\rangle_2)$ in the case where these two possibilities interfere. Then passing through PBS-3, the state will change to $1/\sqrt{2}(|R\rangle_A|L\rangle_B + e^{i\phi}|L\rangle_A|R\rangle_B)$. So when we move the delay mirror adjusting the phase ϕ , the coincident count between D_A^H and D_B^H will oscillate as a function of the position. (b) Experimental result of the oscillation. We can estimate the overlap position from the best fit of the envelop with Gauss function.

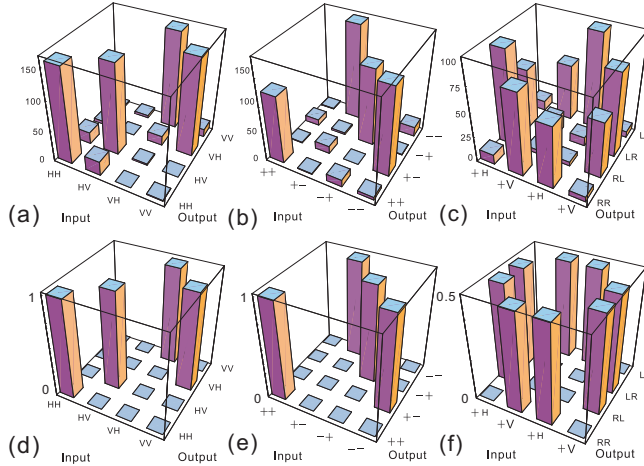


FIG. 3: (color online). Experimental evaluation of the CNOT gate, each data point is measured in 320 min for the first three figures. (a) in the computational basis ($|H\rangle/|V\rangle$). (b) in the complementary basis ($|+\rangle/|-\rangle$). For (c), the input control qubit is in the $|+\rangle/|-\rangle$ basis and the input target qubit is in the $|H\rangle/|V\rangle$ basis, while the output qubits are measured in the $|R\rangle/|L\rangle$ basis. (d),(e) and (f) are theoretical values (for the vertical axis, probability is adopted instead of count rate) for (a), (b) and (c), respectively.